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A function arises when one quantity depends on another. Many everyday relationships between variables can be expressed as functions.

Example Consider the volume of a cylindrical glass with radius = 1 inch. We have a formula for the volume; $V = \pi r^2 h = \pi h \ \ in^3$, where h is the height of the glass in inches and $r = 1$ is the radius.
We see that the value of the volume depends on the height, h; V is a function of h. We sometimes indicate that the value of V depends on the value of h, by writing the formula as

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Piecewise Defined functions

Example The cost of (short-term) parking at South Bend airport depends on how long you leave your car in the short term lot. The parking rates are described in the following table:

First 30 minutes	Free
31-60 minutes	$2
Each additional hour	$2
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- We can give a formula for the cost in terms of time, piece by piece(no single formula):

$$C(t) = \begin{cases} \$0 & 0 \leq t \leq 0.5\ hr. \\ \$2 & 0.5\ hr. < t \leq 1\ hr. \\ \$4 & 1\ hr. < t \leq 2\ hr. \\ \$6 & 2\ hr. < t \leq 3 hr. \\ \$8 & 3\ hr. < t \leq 4\ hr. \\ \$10 & 4\ hr. < t \leq 5\ hr. \\ \$12 & 5\ hr. < t \leq 6\ hr. \\ \$13 & 6\ hr. < t \leq 24\ hr. \\ \$13 + \text{cost of towing} & t > 24\ hr. \end{cases}$$

Definition of Function

Definition A function f is a rule which assigns to each element x of a set D, exactly one element, $f(x)$, of a set E. The **Domain** of a function f is the set D, the set of all values of x, for which $f(x)$ is defined. The **Range** of the function f is the set of all elements of the target set E which have the form $f(x)$, for some x in D.

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- **algebraically**(by an explicit formula).

Algebraic Representation, Domain, Range

All of the functions that we will consider in this course will have a numerical domain and range. Most can be described by giving a a formula for $f(x)$, as in Ex. 1 and 2.

Given a formula for f, unless otherwise specified, the **domain** is the set of all real numbers for which the formula makes sense $= \{x \in \mathbb{R} | f(x) \text{ exists}\}$.

range: set of all numbers of the form $f(x) = \{f(x) | x \in \text{Domain of } f\}$.

Example Let $f(x) = \frac{x^2}{x-1}$. What is the domain of f?

What is $f(1 + \frac{1}{100})$?

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- $\frac{f(0+h)-f(0)}{h} = \frac{f(h)-f(0)}{h} = \frac{\frac{h^2}{h-1} - \frac{0^2}{-1}}{h} = \frac{h^2}{(h-1)h} = \frac{h}{h-1}$.

Absolute Value Function

Note that $f(x) = |x|$ is really a piecewise defined function with formula:

$$f(x) = \begin{cases} -x & x \leq 0 \\ x & x > 0 \end{cases} .$$

Example Write $h(x) = |x - 1|$ as a piecewise defined function.

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- Replacing x by $(x - 1)$ in the definition of the absolute value function above, we get

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- Working through the details, we get

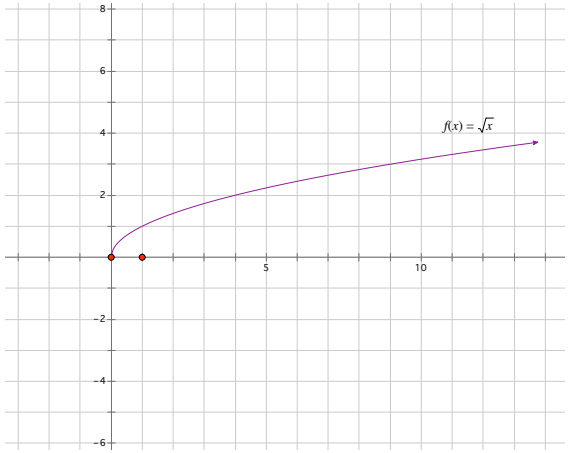
$$h(x) = |x - 1| = \begin{cases} 1 - x & x \leq 1 \\ (x - 1) & x > 1 \end{cases} .$$

Graphs on the Cartesian Plane.

(Please review, graphing and functions in your online homework.)
When we give a formula, for a function of x, $f(x)$, we can **associate an equation** to the function $y = f(x)$.
x is a variable, called the **independent variable**.
y is called a **dependent variable.**
The graph of a function f, with associated formula $f(x)$, is the set of all points (x, y) on the xy-plane that satisfy the equation $y = f(x)$. This is just a picture of all points of the form $(x, f(x))$ on the plane.
The **domain** of a function can be identified from the graph as the set of all values of x for which a vertical line through x cuts the graph (the values of x on the graph).
The **range** of a function can be identified from the graph as the set of all values of y for which a horizontal line through y cuts through the graph (the values of y on the graph).
You should already be very familiar with the graphs shown in the catalogue at the end of the lecture.

$y = \sqrt{x}$

Example If $f(x) = \sqrt{x}$, use the graph of the function to identify the domain and range of f?



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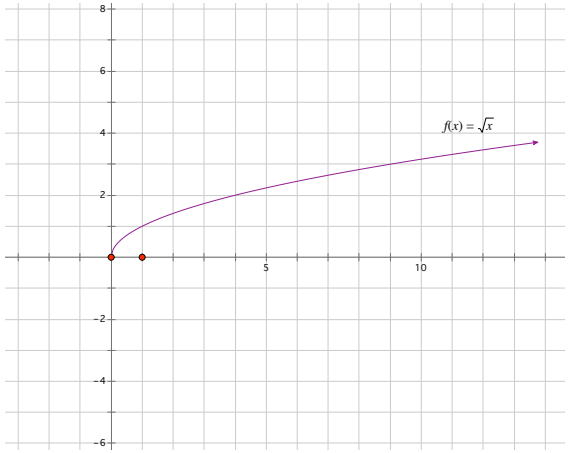

- Domain = set of x values on the graph = $\{x|x \geq 0\}$.

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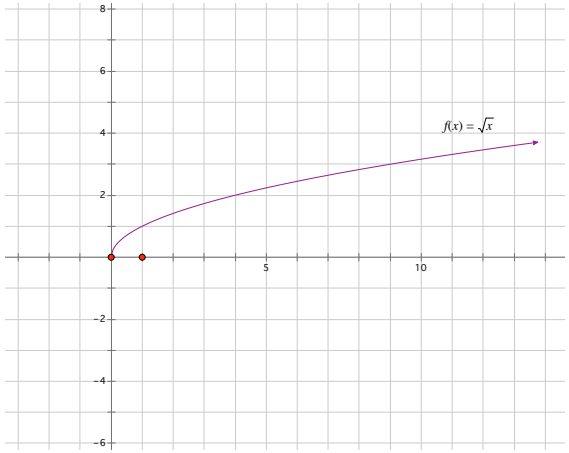
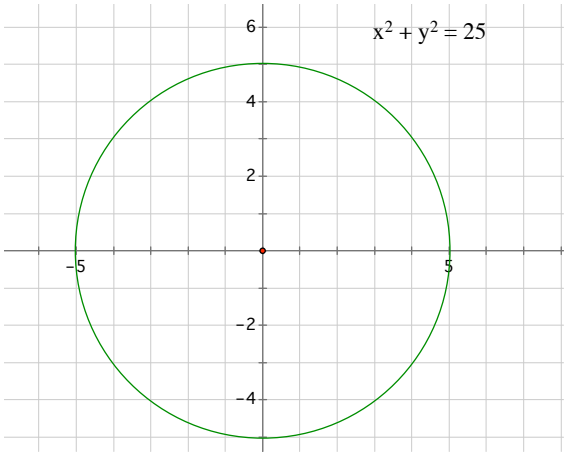


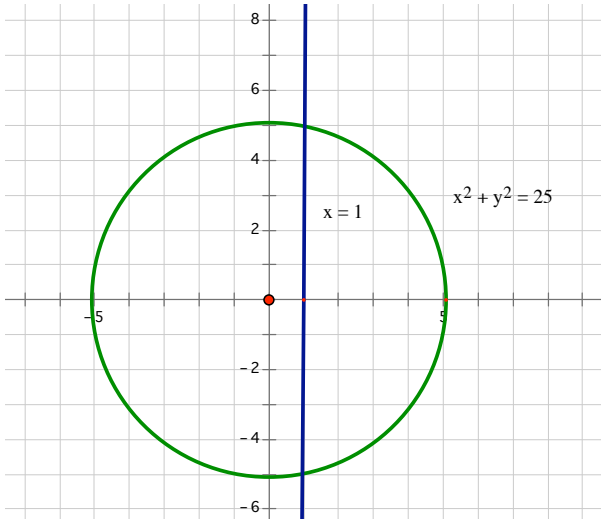
- Domain = set of x values on the graph = $\{x|x \geq 0\}$.
- Range = set of y values on the graph = $\{y|y \geq 0\}$.

Graphing general equations

Graphing general equations : We can plot the points on any curve defined by an equation in x and y on the xy-plane. However, we cannot always solve for y uniquely in such equations and the graph may not be the graph of a function of the form $y = f(x)$.

Example the graph of the equation of a circle of radius 5 with center at $(0, 0)$ is shown on the left below:

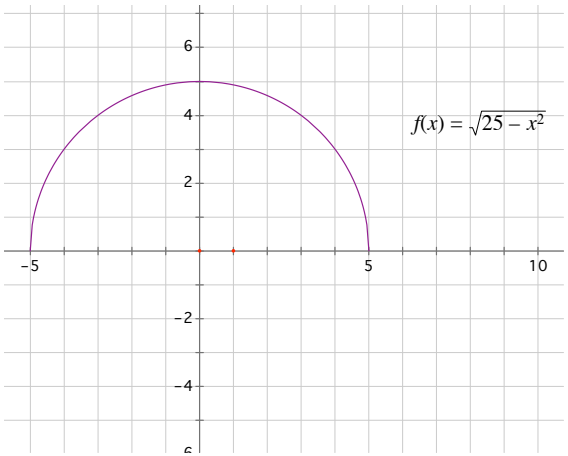


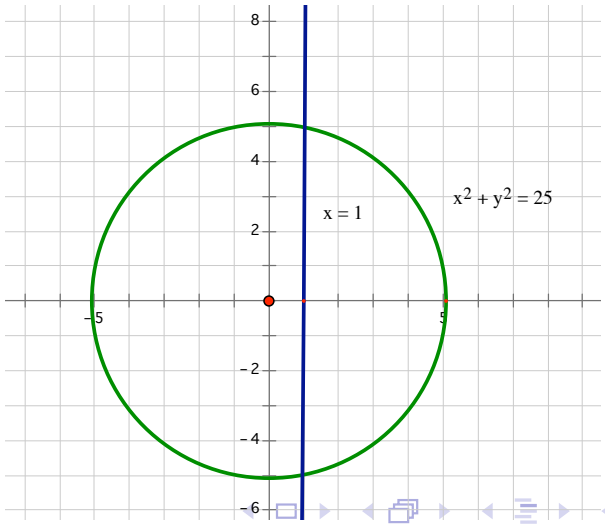


Vertical Line Test

Vertical Line Test (VLT) The graph of an equation passes the vertical line test if each vertical line cuts the graph at most once.

If the graph of an equation passes the Vertical Line Test (VLT) then it is the graph of a function (to each x-value, there corresponds at most one y-value on the graph), if we can solve for y in terms of x, we will get the equation corresponding to the function: $y = f(x)$.





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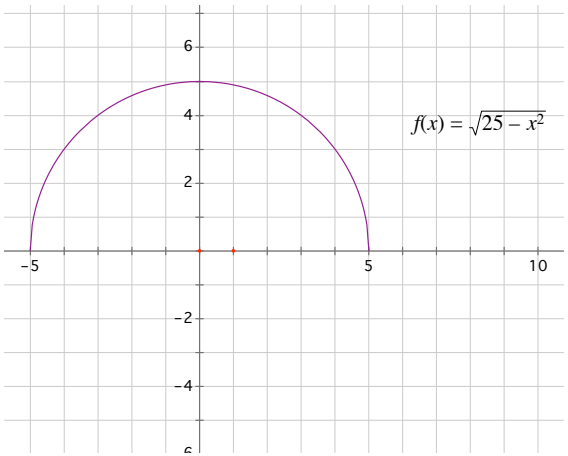
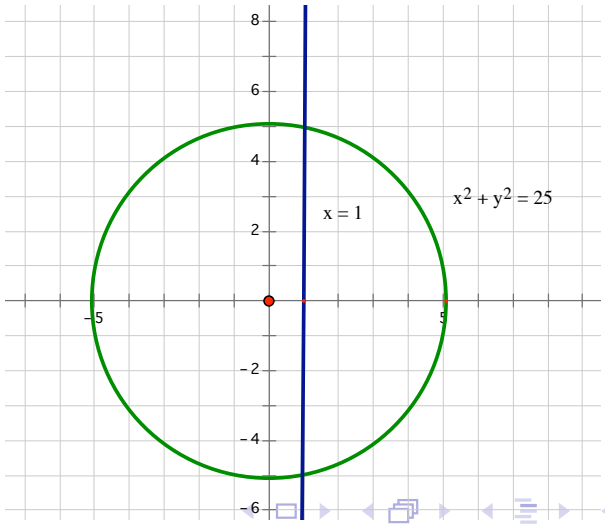
- Clearly the graph of the equation $x^2 + y^2 = 25$ does not pass the vertical line test (see the vertical line drawn in the picture on the right below)





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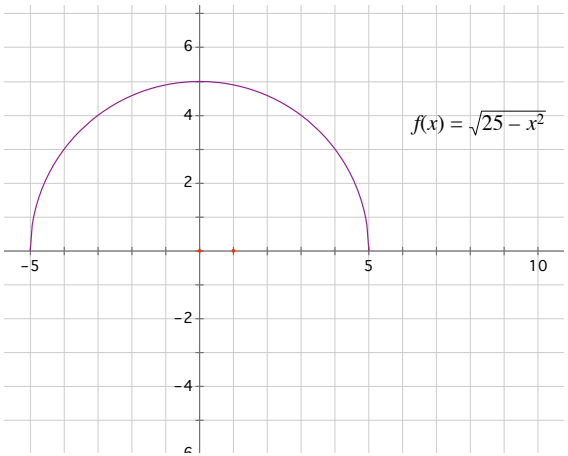
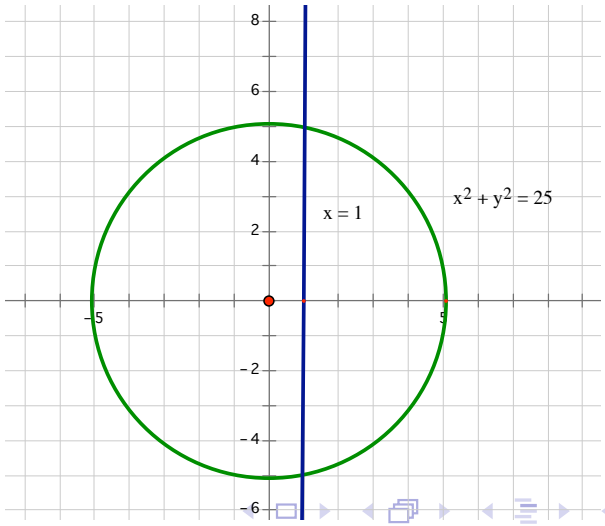
- Clearly the graph of the equation $x^2 + y^2 = 25$ does not pass the vertical line test (see the vertical line drawn in the picture on the right below)
- On the other hand the upper half of the above circle does pass the VLT and is the graph of the function, $f(x) = \sqrt{25 - x^2}$.





Graphing Techniques

We can use our small catalogue of graphs from the appendix to graph many functions, by taking note of how graphs change with the transformations listed below:

Vertical and Horizontal Shifts

VERTICAL AND HORIZONTAL SHIFTS Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Example Sketch the graph of the equation $y = x^2 - 6x + 14$.

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Example Sketch the graph of the equation $y = x^2 - 6x + 14$.

- $y = x^2 - 6x + 14 = \left[x^2 - 6x + \left(\frac{-6}{2}\right)^2\right] - \left(\frac{-6}{2}\right)^2 + 14$

Graphing Techniques

We can use our small catalogue of graphs from the appendix to graph many functions, by taking note of how graphs change with the transformations listed below:

Vertical and Horizontal Shifts

VERTICAL AND HORIZONTAL SHIFTS Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

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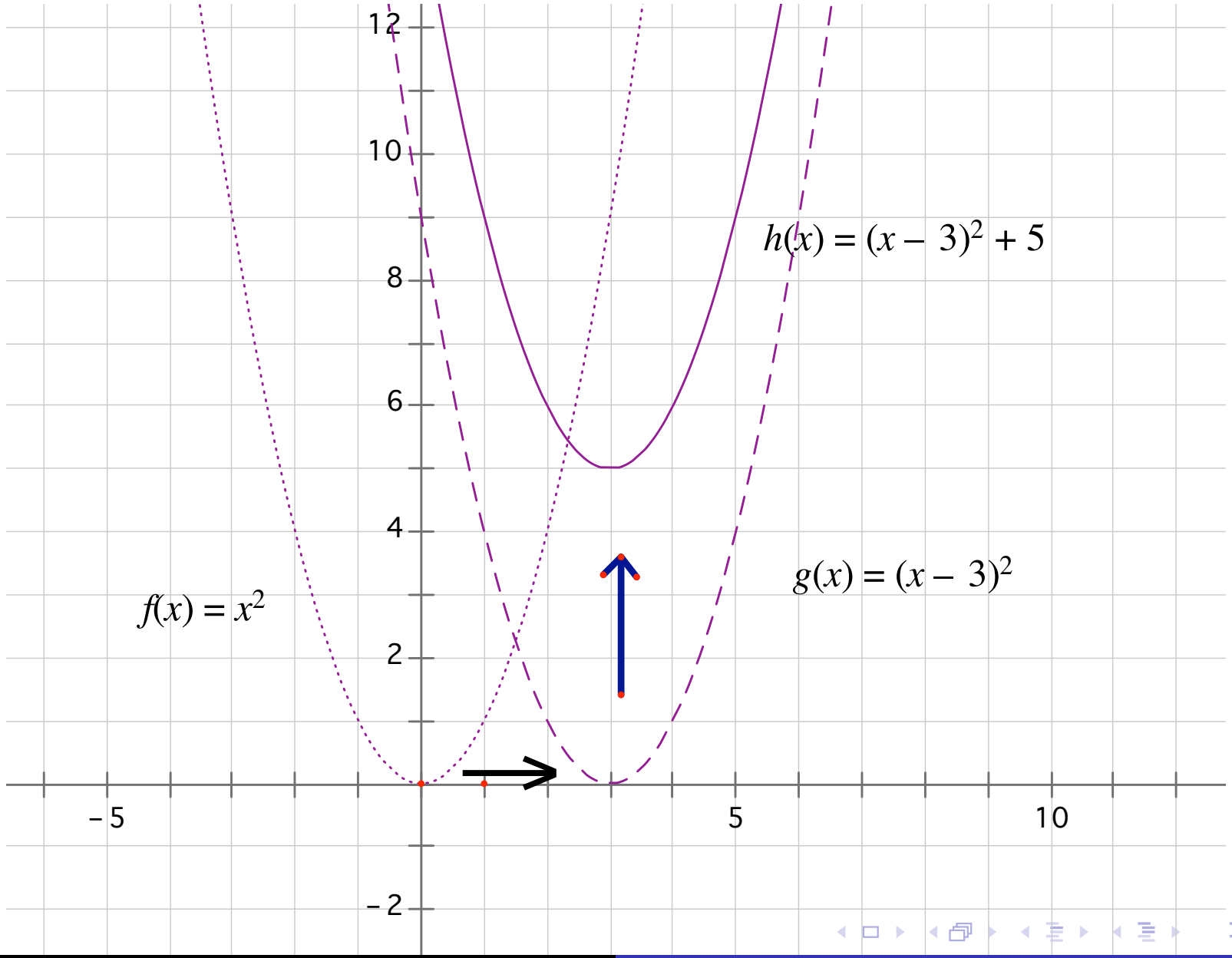
- $= [x - 6x + 9] + 5 = (x - 3)^2 + 5$
- To graph this equation, we shift the graph of $y = x^2$, first by three units to the right and then five units upwards.

$y = (x-3)^2 + 5$



Stretching, Compressing and Reflecting Graphs

VERTICAL AND HORIZONTAL STRETCHING AND REFLECTING Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

$y = (1/c)f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c

$y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c

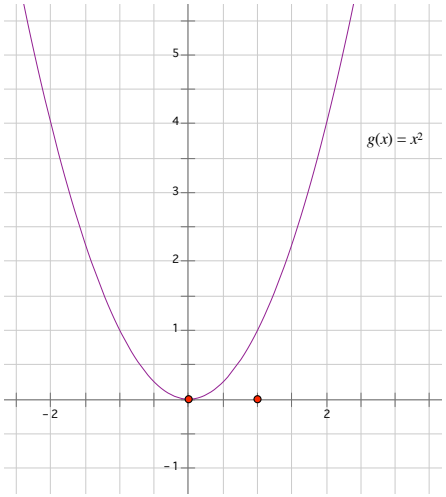
$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis

In the Appendix, it is demonstrated how to use these techniques to graph $y = 2\sin(4x)$.

Increasing/Decreasing Functions

Definition We say a function $f(x)$ is increasing on the interval $[a, b]$, if the graph is moving upwards from left to right. Algebraically, this amounts to the statement:
A function f is called **increasing** on the interval $[a, b]$ if $f(x_1) < f(x_2)$ for any two numbers x_1 and x_2 with $a \leq x_1 < x_2 \leq b$.
Similarly we say the function $f(x)$ is decreasing on the interval $[a, b]$ if the graph of f is moving downwards from left to right on that interval
or A function f is **decreasing** on the interval $[a, b]$ if $f(x_1) > f(x_2)$ for any two numbers x_1 and x_2 with $a \leq x_1 < x_2 \leq b$.
Example Use the graph of $y = x^2$ to find the intervals where f is increasing and decreasing.



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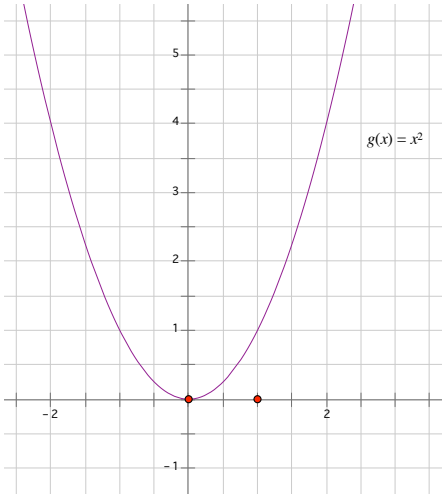

- The graph is decreasing on the interval $(-\infty, 0]$.

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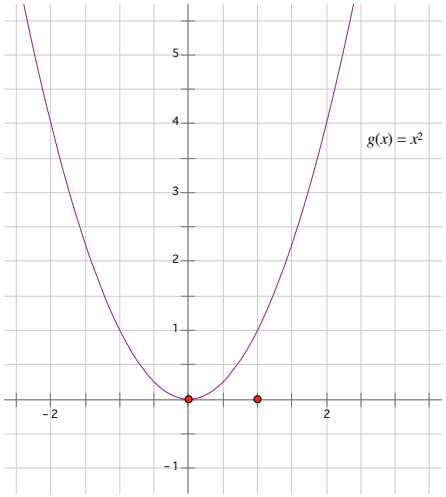

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Operations on Functions

As well as being able to add, subtract, multiply and divide functions, we can form the composition of two functions f and g.

Definition Given two functions f and g, the composite function $f \circ g$ is defined by

$$(f \circ g)(x) = f(g(x)).$$

The domain of $f \circ g$ is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f

It is implicit in the definition of the function, that we first calculate $g(x)$ and then apply f to the result. Hence any x in the domain must satisfy the above two conditions.

(When calculating the domain, calculating just the formula for $f \circ g$ or $g \circ f$ can be misleading)

$(f \circ g)(x)$

Example If $f(x) = \sqrt{x}$ and $g(x) = x^2 + 1$,
then $(f \circ g)(x) = f(g(x)) = f(x^2 + 1) = \sqrt{x^2 + 1}$.
On the other hand, $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = (\sqrt{x})^2 + 1 = x + 1$.
The domain of f is $\{x \in \mathbb{R} | x \geq 0\}$ and the domain of g is the set of all real numbers.
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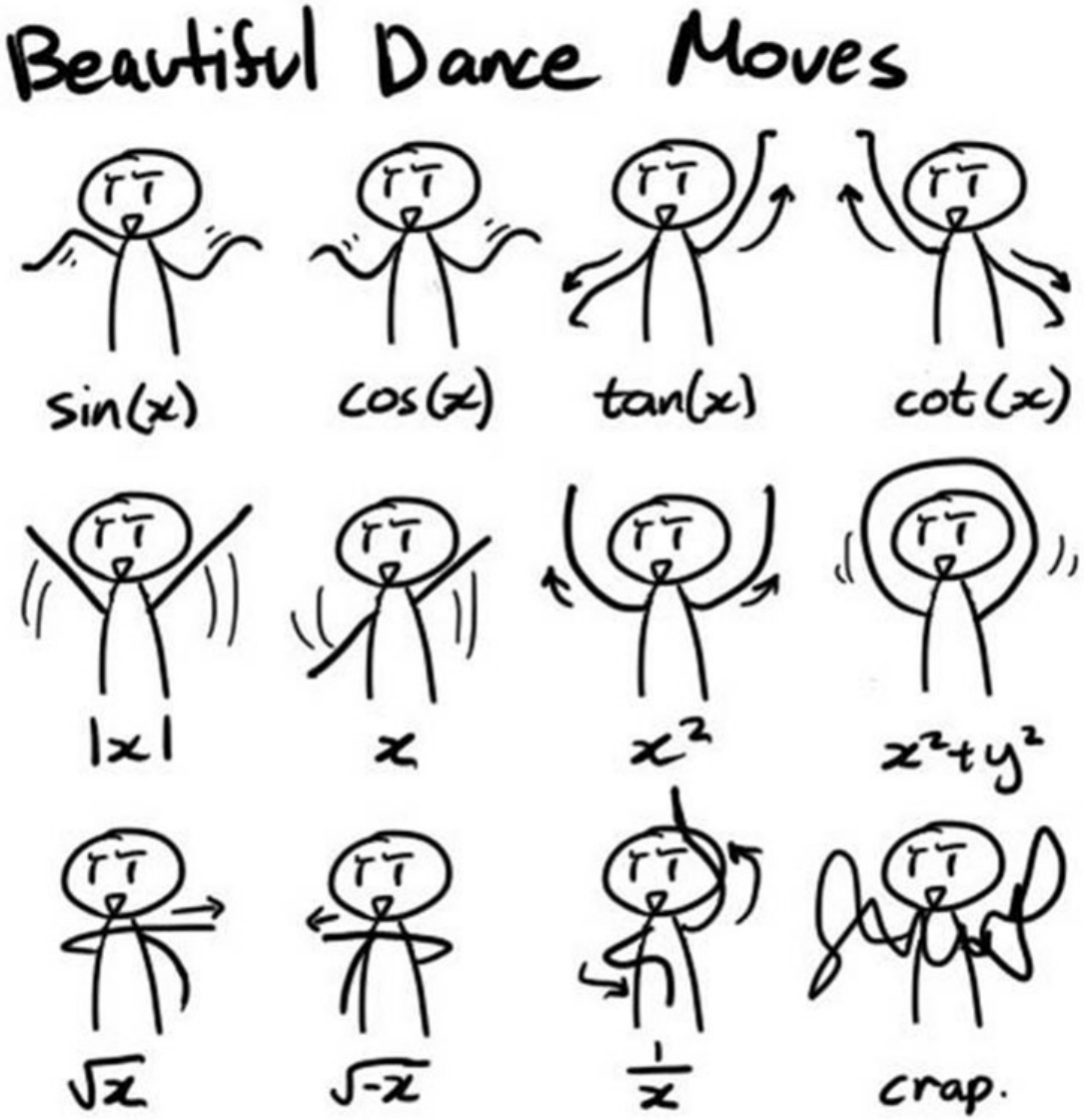
Functions as Dance Moves